Markov Modeling of Energy Harvesting Body Sensor Networks

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***Abstract*—The emerging paradigm of wearable and implantable medical sensors has enabled continuous and unobtrusive monitoring for patients and human subjects, allowing them to continue their normal activities, and yet be assured of immediate response in case of a detected health emergency. Energy harvesting has been proposed as a viable scheme for powering such sensors as periodic retrievals for battery replacements may not be feasible. The current state of the art in energy harvesting allows tapping into several physical and naturally existing sources, such as solar, wind, vibration, RF scavenging, among others. However, there is a lack of theoretical models that can predict future consumption and residual availability of energy in a sensor node equipped with multiple boards that can simultaneously operate on different types of sources. In this paper, we propose MAKERS, a Markov model based method to capture the energy states of such sensors. MAKERS allows detailed prediction of the probability of a node failing to detect an event owing to lack of energy, which is a key design consideration for body sensor sensors.**

***Index Terms*—Body Sensor Networks; Energy Harvesting; Self-Powered Networks; Markov Processes**

I. INTRODUCTION

The number of applications of Body Sensor Networks (BSNs) on health monitoring is increasing exponentially during the last years. Real time wearable sensors and actuators can collect useful medical data and communicate with off-body networks [1]. Various application scenarios for BSNs have been proposed, including sensing vital parameters of patients suffering from chronic diseases, sports medicine, soldier and warfighter health monitoring, security, among others.

However, the major obstacle for adoption of BSNs is the energy supply. Batteries do not provide enough energy to maintain sensor nodes during long periods and other alternatives have to be considered. Energy harvesting (EH) is a candidate solution to bridge the energy gap; devices enabled with this technology can harvest energy from a number of natural and artificial sources for sustained network operation. While EH has already been demonstrated in BSNs [2] and [3], environmental conservation and cost savings associated with fewer battery replacements can be considered as complementary benefits of this technology.

The goal of EH networks is to ensure continuous network operation, wherein nodes may regulate their transmission and harvesting cycles judiciously to meet network constraints. Specifically, in this paper, we provide a a discrete time model of a given wearable sensor that senses a desirable parameter and reports the data to a sink, which is an external device. The model, called as MAKERS (Multiple boArd marKov modEl for Energy haRvesting Sensors) integrates the energy model and the traffic model of the sensor nodes, and allows us to analyze the overall system to obtain performance metrics.

Only recently has the research community engaged in developing higher layer network protocols for EH networks. Despite these strides in the area of protocol design for EH networks, the development of theoretical models for energy harvesting, and the prediction of the residual energy state during an ongoing network operation are still in a nascent stage. In [4], authors try to analyze the average energy harvested from a board; in [5] and [6], researchers review some model of energy harvesting nodes using Markov processes.

Analytical formulations modeling both the energy status of a node and the traffic model for the case of a single harvesting source have been previously presented in [7] [8], which we extend significantly in this paper as follows:

- We develop a Markov model for capturing the energy states of the sensors equipped with multiple energy harvesting boards. These sensors can harvest energy from the same source, or a combination of different sources (such as vibration and RF). This presents a general case for EH BSN design, as a single source cannot be assumed to be always present during network operation.
- We provide simplified analytical models for predicting the probability of a sensor running out of energy (hence, mis-detecting the event, which we call as the *event-loss*). Compared to earlier work [7] [8], not only are our models lower in complexity facilitating on-board computation in the sensors, but can also be applied for sensors with multiple harvesting boards.

The rest of this paper is described as follows: In Section II, we list the general assumptions taken in the model. In Section III we develop the MAKERS model. We derive analytically the *event-loss* probability in Section IV. In Section V, we adapt the model to work with multiple energy sources, with a thorough performance evaluation in Section VI. Finally, in Section VII, we conclude our work.

II. Preliminary Discussion and Assumptions

The MAKERS model assumes the case of multiple boards, capable of harvesting energy from different sources. Before we present the general model, we develop our analysis under the constraint that all the M EH boards connected to each sensor harvest energy from the same source a (e.g. two shoe-mounted piezoelectrics). Later, we relax this constraint to account for multiple sources. Note that many of the notations used in this section are similar to the earlier work in [7].

Each EH board (out of the total M) can be independently in the *active* (i.e., currently harvesting with a rate ρ_a) or in the *inactive* (off) state. Further, the time duration for which the sensor stays in these two states are exponentially distributed with the means T_{on} and T_{off}, respectively. The probability of changing from *active* to *inactive* is r, and the reverse, is w. Hence, the overall probability of the EH board to be *active* is:

$$\mu = \frac{w}{r+w} \tag{1}$$

An event that needs to be sensed and reported occurs with a probability equal to p. The time between these sensing events, t_p, is exponentially distributed with mean T_p. Each event consumes a total amount of energy equal to E, which includes the energy expended nodal processing, as well as transmission or reception of the data. If there is no event during a time slot, no energy is consumed (the on-board sensors are passive components), and battery leakage is negligible.

We note that the required energy E for a given sensing event may not be completely harvested within a single time unit T. Thus, multiple slots of duration T may be needed to obtain sufficient energy within the node, which is given as $k = \frac{E}{\rho_a T}$ [8]. Let $i \in [0, M]$ be the number of *active* boards in a time slot. Without loss of generality, we assume that $1 \leq M \leq k$, where M is a positive integer and k a positive real number. In addition, the node incorporates a battery or a super-capacitor, with a storage capacity equal to $(N-1)E$. Hence, a full battery will allow us to run $N-1$ events.

The time unit or slot used in this model is T, where $T \ll T_{on}, T_{off}, T_p$. Hence, only one event can occur in a time slot. As derived earlier in [8], r can be expressed as: $r = \frac{T}{T_{on}} e^{-\frac{t}{T_{on}}} \approx \frac{T}{T_{on}}$. Therefore, the probabilities w and p are $w \approx \frac{T}{T_{off}}$ and $p \approx \frac{T}{T_p}$, respectively. Note that while distributions other than the exponential are possible for the event intervals, the meaning of the variables r, w and p in the overall system model will remain unchanged.

III. The MAKERS Model

In this section, we develop our proposed analytical model. Our approach will be to start first with a finer granularity of a $(M+1)kN$ states model, that also splits up each inter-sensing event duration into k sub-states (recall k fractional harvesting durations give the energy required for the sensing event). Then, by merging the sub-states every k, we build the general $(M+1)N$ model. Finally, we describe how $M+1$ merge into 1 and establish a simplified version of the model with N states, which shall help in the following sections.

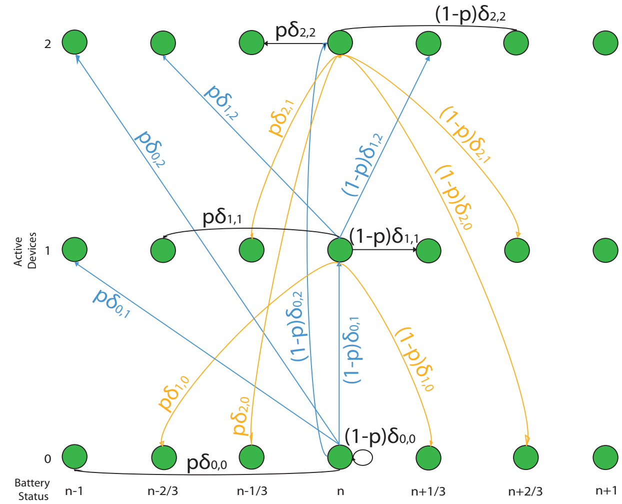

Fig. 1. $(M+1)kN$ states model, $M=2$ and $k=3$ centered at state n

The MAKERS general model has $(M+1)N$ states, a product of the $M+1$, from 0 to M *active* harvesting boards, and N, which represents the amount of energy left in the battery in the current time slot. During a time slot, the node harvests a total amount of energy equal to $i\rho_a T$ and spends E if an event occurs. Moreover, the energy that is being harvested during the current time slot can not be used to run an event that happens in it, unlike the assumptions made in [7]. In other words, if we have residual energy $\frac{k-1}{k}E$ in the battery, we will not be able to run an event during this time slot even though many of the boards are *active*.

A. The $(M+1)kN$ states model

Firstly, we focus on how the system changes from one state to another with respect to the battery life. In figure 1, we show the Markov chain for the $(M+1)kN$ model in the case of $M=2$ and $k=3$, with the transition probabilities from the states having the same energy n. Each horizontal row of states corresponds to a number of active boards (hence, we have three rows for $M=2$), while the vertical columns are energy sub-states, with three columns ($k=3$) between the two residual energy state n and $n+1$. Let $\delta_{i,j}$ be the probability of j harvesters *active* in the future state if i harvesters are *active* in the current one.

For example, if we are in state with residual energy n and 1 *active* board ($2nd$ row, $4th$ col.) and an event occurs, the node will consume E at the same time that harvests $\frac{1}{3}E$, so the future state will have a residual energy equal to $n-\frac{2}{3}$. Then, 3 possible transitions may occur depending on the future number of *active* boards, described by $\delta_{1,j}$. For the states corresponding to the special cases of the battery completely *full* or *empty*, the transition probabilities are slightly different to the ones presented, and discussed later in this section.

B. The $(M+1)N$ states model

To create the simplified $(M+1)N$ model, every k energy levels will be merged into one. For example, the first k states

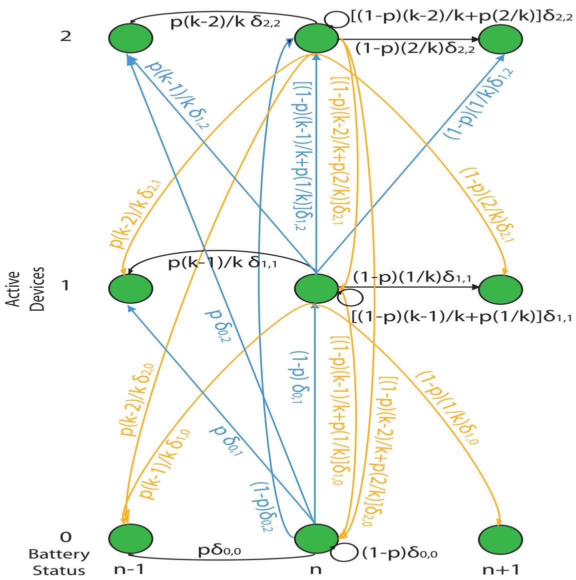

Fig. 2. (M+1)N states model, M=2, generic k focused in n

in terms of *battery life*, called $0, \frac{1}{k}, \ldots, \frac{k-1}{k}$; will be merged in a single state representing the residual energy 0. From figure 1 we can formally define the transition probabilities $p_{a,b/i,j}$, where a is the current energy level, b the future energy level, and the number of current and future *active* boards is given by i and j, respectively:

- $p_{n,n-1/i,j} = p\frac{k-i}{k}\delta_{i,j}$
- $p_{n,n/i,j} = [(1-p)\frac{k-i}{k} + p\frac{i}{k}]\delta_{i,j}$
- $p_{n,n+1/i,j} = (1-p)\frac{i}{k}\delta_{i,j}$

Note that from an arbitrary state n we can only reach another state with a battery level that differs by a single unit, i.e., only states with battery level equal to $n-1$, n and $n+1$ are possible, with other transition probabilities set to 0. Moreover, if an event happens in state n we will not be able to reach the state $n+1$ (higher energy). Similarly, $n-1$ (lower energy) can only be reached if an event occurs. In figure 2 we can see the Markov chain for the $(M+1)N$ model for $M = 2$ and a generic k with the transition probabilities from the states with energy n represented. Again, the two boundary states with energy level 0 and $N-1$ will have different probabilities for the following cases:

- $p_{0,0/i,j} = \frac{k-i}{k}\delta_{i,j}$
- $p_{0,1/i,j} = \frac{i}{k}\delta_{i,j}$
- $p_{N-1,N-1/i,j} = [(1-p) + p\frac{i}{k}]\delta_{i,j}$

In order to obtain $\delta_{i,j}$, i.e., the probability of j harvesters being *active* from an earlier total number i, we take into account all the possible combinations of *active-inactive* changes that may happen. We observe that there are two possibilities:

First, when $j \leq i$, $i-j$ boards need to change their state from *active* to *inactive* while the rest remain in the same state. However, many other possibilities can occur because some other currently *active* boards can switch to *inactive* if the same number of *inactive* boards switch as well. Similarly, when j is bigger than i, we have to take into account the different combinations, all of which have finite occurrence probability.

Owing to space constraints, we directly state the expressions for $\delta_{i,j}$ for the above two cases $i < j$ and $j \leq i$, which are obtained through straightforward but tedious analytical derivations:

- $0 \leq j \leq i$:

$$\delta_{i,j} = \sum_{l=0}^{\min(M-i,j)} \binom{i}{j-l}\binom{M-i}{l}(1-r)^{j-l} r^{i-(j-l)}(1-w)^{M-i-l}w^{l} \qquad (2)$$

- $i < j \leq M$:

$$\delta_{i,j} = \sum_{l=0}^{\min(M-j,i)} \binom{i}{l}\binom{M-i}{M-j-l}(1-r)^{i-l} r^{l}(1-w)^{M-j-l}w^{j-(i-l)} \qquad (3)$$

Once these state transition probabilities are calculated, the MAKERS model is completely represented. Next, we describe a simplified version of this model, which shall help in obtaining closed form equations about the performance of the device.

C. Simplified N states model

Here, the state only denote the battery status, from 0 to $N-1$, individual boards' states do not appear in the state definition. In every energy level, the case of having any arbitrary number of *active* boards is considered. Recall that the probability of a board to be *active*, given by equation 1, the probability of having i *active* boards, ϕ_i, is:

$$\phi_i = \binom{M}{i}\mu^i(1-\mu)^{M-i} \qquad (4)$$

The sum of transition probabilities from one energy state to another, or to the same state (states defined in terms of energy level alone) remain the same as the general MAKERS model in Section III-B. Hence, we only take into account the number of *active* boards in the current state, which leads us to a very simple model. We define α, a variable that we use subsequently, as follows:

$$\alpha = \sum_{i=0}^{M}\frac{i}{k}\phi_i \qquad (5)$$

From figure 3, we can express the new transition probabilities, $p_{a,b}$, where a is the current energy level, b the future one, as follows:

- $p_{n,n-1} = p(1-\alpha)$
- $p_{n,n} = (1-p)(1-\alpha) + p\alpha$
- $p_{n,n+1} = (1-p)\alpha$

Using this simplified expression, we can now compute the closed form expression of the probability of *event-loss*.

IV. Probability of Event-Loss

An event is lost (i.e. is not sensed and reported), if a node lacks sufficient energy E to process it. This probability, P_L, can be found solving the equation system defined by $\mathbf{P}\pi = \mathbf{P}$ and $\sum_{n=0}^{N-1}\pi(n) = 1$, where $\mathbf{P}$ is the transition probability

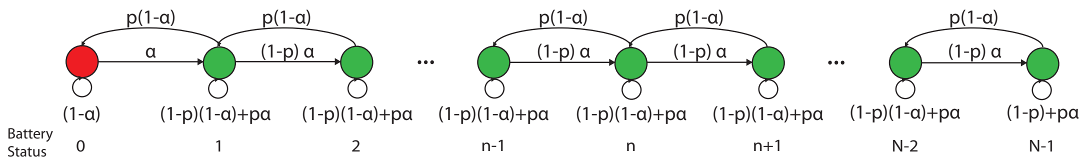

Fig. 3. Simplified N states model

matrix, π is the vector of the steady probabilities for each state, from 0 to $N-1$; being $P_L = \pi(0)$, the loss probability. The simplified Markov chain model allows us to derive a closed form solution analyzing the equilibrium equations for different states. We begin from the state 0, which can be written as:

$$\pi_0 \alpha = \pi_1 p(1-\alpha) \tag{6}$$

We continue with state 1:

$$\pi_1(p(1-\alpha) + (1-p)\alpha) = \pi_0 \alpha + \pi_2 p(1-\alpha) \tag{7}$$

If we replace π_0 using equation 6, we find:

$$\pi_1 = \frac{p(1-\alpha)}{(1-p)\alpha}\pi_2 \tag{8}$$

Hence, for an arbitrary state n, $1 < n < N-1$:

$$\pi_n(p(1-\alpha)+(1-p)\alpha) = \pi_{n-1}(1-p)\alpha + \pi_{n+1}p(1-\alpha) \tag{9}$$

This time, we analyze it in the case of $n = 2$:

$$\pi_2(p(1-\alpha) + (1-p)\alpha) = \pi_1(1-p)\alpha + \pi_3 p(1-\alpha) \tag{10}$$

Now, we use equation 9 to substitute π_1, therefore:

$$\pi_2 = \frac{p(1-\alpha)}{(1-p)\alpha}\pi_3 \tag{11}$$

Notice that expression for π_2 is similar to 8, for π_1. Continuing the analysis for state $N-1$:

$$\pi_{N-1} p(1-\alpha) = \pi_{N-2}(1-p)\alpha \tag{12}$$

Observing the similar pattern as seen in earlier equations 8, 11 and 12, we express π_{n+1}, $1 \leq n < N-1$, as:

$$\pi_{n+1} = \frac{(1-p)\alpha}{p(1-\alpha)}\pi_n \tag{13}$$

We define γ as follows, to simplify future equations:

$$\gamma = \frac{(1-p)\alpha}{p(1-\alpha)} \tag{14}$$

Combing the equation 13 and the one for state 0, eq. 6, we rewrite π_n, $1 \leq n \leq N-1$, as:

$$\pi_n = \frac{\gamma^n}{(1-p)}\pi_0 \tag{15}$$

The last step is to find π_0, or P_L. We know that the sum of all the steady state probabilities must be equal to 1. Therefore, we can re-write 6 as follows:

$$\begin{gathered}\pi_0(1 + \textstyle\sum_{n=1}^{N-1} \frac{\gamma^n}{(1-p)}) = 1 \\ \pi_0(1 + \frac{(1-\gamma)}{(1-p)(1-\gamma)} \textstyle\sum_{n=0}^{N-1} \gamma^n - \frac{1}{(1-p)}) = 1\end{gathered} \tag{16}$$

We could insert the following form for the series in the previous equation 16, with the constrain $0 < \gamma < 1$, so $0 < (1-p)\alpha < p(1-\alpha)$.

$$(1-\gamma)\sum_{n=0}^{N} \gamma^n = 1 - \gamma^{N+1} \tag{17}$$

Finally, we will have two cases for P_L depending on the constraint stated in the last equation:

$$P_L = \begin{cases} \frac{(1-p)(1-\gamma)}{1-\gamma^N - p(1-\gamma)} & (1-p)\alpha < p(1-\alpha) \\ \frac{1}{1+\frac{1}{1-p}\sum_{n=1}^{N-1}\gamma^n} & (1-p)\alpha > p(1-\alpha) \end{cases} \tag{18}$$

Returning to the $(M+1)N$ model with its steady state probabilities, $\Pi_{n,i}$, where n is the energy level and i the number of active boards for $0 \leq n \leq N-1$ and $0 \leq i \leq M$, we compute:

$$\Pi_{n,i} = \phi_i \pi_n \tag{19}$$

We observe that these close forms are much simpler that the ones presented in [7], which account for the case of $k = 1$ and only one source of energy harvested.

V. Model for many different sources

In our harvesting model, energy from different sources can be obtained by the same node. Therefore, we introduce a modification of MAKERS in order to work with boards with different features, in terms of average rate of energy harvested and probabilities to switch their state. We first address a simplified case of two different sources, to then discuss how can be extended further.

A. Model for two different sources

For the two source model, following the analysis in Section III with the same notation, we have:

- *Board A*: ρ_A, r_A, w_A, $\mu_A = \frac{w_A}{r_A + w_A}$
- *Board B*: ρ_B, r_B, w_B, $\mu_B = \frac{w_B}{r_B + w_B}$

We define ρ_B as $\rho_B = b\rho_A$, where b is a real positive number. We keep the definition of the parameters of the model, but with one new constraint of $b + 1 \leq k$. Applying this new model, the parameter α is re-visited, giving a new form that will let us compute the derived formulae:

$$\alpha = \frac{1}{k}\mu_A(1-\mu_B) + \frac{b}{k}(1-\mu_A)\mu_B + \frac{b+1}{k}\mu_A\mu_B \tag{20}$$

Here, the first term of the summation represents the case when only board A is *active*, the second when only board B is *active*, and finally, the third term gives the case for both boards being *active*. This expression can be trivially extended using

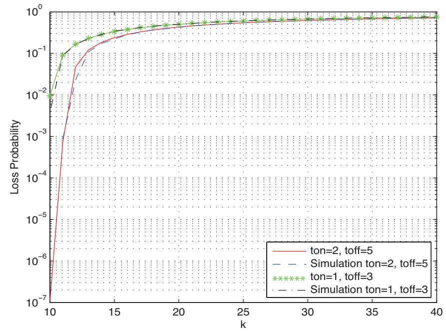

Fig. 4. Loss Probability (k) for N=100, M=2, p=0.05

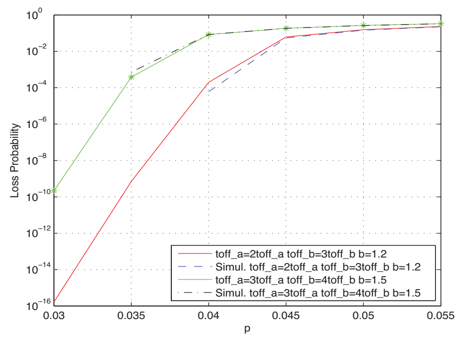

Fig. 5. PL (p) for different sources N=100, k=15

different energy harvesting rates ρ_i for a board i, assuming that ρ_i is a multiple of the base rate ρ_A, i.e., $\rho_i = \psi\rho_A$, $\psi > 0$. Along the same lines, α is easily adapted to consider all the cases of each of the combination of boards being in the *active* state, as shown in the two board example in eq 20.

VI. Results

Monte-Carlo continuous-time simulations are undertaken in MATLAB to evaluate our approach. The values for t_a, t_i and t_p are randomly generated through exponential distributions with means T_a, T_i and T_p, respectively. The values of these parameters are mentioned in the individual figures.

Figure 4 compares the *event-loss* probability obtained from theory, Eq. 18, with those obtained from the simulations. Every harvesting board has its own parameters following exponential distributions, with the same mean time for the state durations, assuming they have the same features. In this figure, we compare the results by varying k. Owing to space constraints, we show the case for $M = 2$.

Figure 5 compares the probability of *event-loss* P_L, depending on p, between theory and simulations, for the case of different sources. Simulations points are not plotted for very low values of p i.e., $p < 0.035$, as we did not observe a statistically significant number of missed events, given the simulation time and the values assumed for the other parameters. However, we

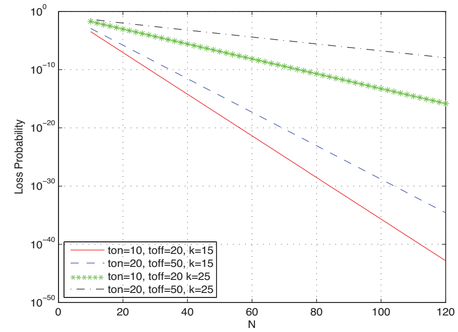

Fig. 6. Loss Probability (N) for M=2, p=0.02

observe a good match in other scenarios.

Figure 6 presents the P_L, eq. 18, versus N, for different k, w and r. Our model could identify battery specifications, depending on desired loss probability we want to achieve and the other device-specific parameters of the system.

VII. Conclusions and Future Work

In this paper we have presented MAKERS, a Markov based model for multiple-sources energy harvesting nodes in WSNs. It considers both the number of the harvesting boards attached to the node, as well as the remaining energy of the battery to determine the state of the node. A closed form solution for the *event-loss* probability has been derived. The results of our simulations are in good agreement with the derived closed for these parameters, thereby verifying the accuracy of our approach. Through our prediction models, the network designer can set the requirements for sensor nodes with many EH-boards, such as the battery capacity. Our future work will be focused on practical comparisons with real sensors with multiple boards with the predictions of the MAKERS model.

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